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Vacuum energy and dynamical symmetry breaking in curved spacetime

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Abstract: We argue that calculating vacuum energy requires quantum field theory whose axioms are adapted to curved spacetime. In this context, we suggest that non-zero vacuum energy is connected to dynamical breaking of electroweak symmetry. The observed meV scale can be understood in terms of electroweak physics via a naive estimate. The scenario requires all particle masses to have a dynamical origin. Any Higgs particle has to be a composite, and the origin of vacuum energy might be probed at the LHC.

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The vacuum energy problem. Vacuum energy has a long history [1, 2], and it has been brought to focus by cosmological observations which have in the past two decades established that the universe expands faster than expected. The usual view of the vacuum energy problem[1] is that quantum field theory (QFT) predicts vacuum energy, cosmological observations reveal its presence, and there is a huge discrepancy between the predicted and measured values.

One measure of the perceived severity of the problem is the popularity of anthropic arguments. When no solution that would determine the value of a physical quantity has been found, it may be tempting to take this as evidence that there is no solution, and that the measured value is due to environmental contingency. This road was earlier taken with regard to spatial curvature [3], though anthropic arguments were eventually left by the wayside when a dynamical, testable, explanation was found with the introduction of inflation. The appeal to anthropic arguments indicates that the problem does not involve contradiction between theory and observation, nor theoretical inconsistency, simply unmet expectations. In the vacuum energy case, it is useful to consider these expectations carefully, because the simple summary given above is somewhat misleading.

No vacuum energy in flat spacetime QFT. First of all, the interpretation of the observations in terms of vacuum energy is not beyond reasonable doubt; apart from explanations involving exotic matter or modifications to gravity, it is possible that the change in the expansion rate is simply due to the known breakdown of homogeneity and isotropy at late times [4, 5]. Nevertheless, even if the cosmological observations were explained by something else than vacuum energy, the question of why vacuum energy is not large would remain. Let us assume that the correct explanation for the observations is indeed vacuum energy, and look at the theoretical side.

It is often claimed that the Casimir effect shows that vacuum energy is real [1]. However, as discussed in [6], the Casimir effect can be calculated without any reference to vacuum energy, and gives no more indication of the reality of vacuum energy than any other loop contribution in QFT. Indeed, the Lamb shift was quoted to the same effect in the background material for the 2011 Nobel prize in physics, awarded for the discovery of accelerating expansion [7]. However, while loop effects may be related to vacuum fluctuations, these are distinct from vacuum energy[2].

[1]The terms vacuum energy and cosmological constant are often used interchangeably. The cosmological constant is a geometrical term in the classical Einstein equation, and it changes the way spacetime curves in the absence of matter. Vacuum energy has its origin in quantum theory, and it is a form of matter which has the same effect as the cosmological constant. We only discuss vacuum energy.

[2]In particular, referring to vacuum energy as the energy of fluctuations of the vacuum is misleading, because the vacuum is an energy eigenstate, so there are no fluctuations in the energy.

QFT formulated in flat spacetime is sensitive only to differences in energy: an arbitrary constant can be added to the Lagrangian without changing the physics, so QFTs which differ only by a constant energy are equivalent. As the concept of absolute energy is not part of the physical content of flat spacetime QFT, it is even in principle impossible to obtain any prediction about the value of vacuum energy, any more than it is possible to predict gauge-fixing parameters.

Arguments about vacuum energy in flat spacetime are therefore at most suggestive, and suggestions are easy to misconstrue. For example, it has been argued that calculating vacuum energy by integrating over momentum space modes with a cut-off Λ much larger than the masses present in the theory shows that it is of the order Λ^4. However, the resulting energy-momentum tensor is traceless, so it behaves like radiation and does not lead to accelerating expansion, as is well known [1, 8]. When the integration range is extended to infinity, the equation of state becomes indefinite as the result diverges.

In any case, vacuum expectation values in QFT do not reduce to simple momentum mode decomposition [8]. From the field theory point of view, vacuum energy is a renormalisable term (supersymmetric theories excepted), and its value is arbitrary, like particle masses. However, whereas masses are fixed by observation, vacuum energy is not observable in flat spacetime QFT. If spacetime is curved, the situation is different, since gravity responds to absolute and not relative amounts of energy. The new piece of information which has to be supplied is how quantum fields couple to gravity.

Symmetry breaking in curved spacetime QFT. The effect of quantum fields on spacetime curvature can be studied in semiclassical quantum gravity, where spacetime is treated classically and its dynamics is sourced by the expectation value of the energy-momentum tensor. Vacuum energy then becomes a physical quantity with arbitrary magnitude. However, the vacuum state can evolve as the universe expands. A well-known problem is that changes in the vacuum energy associated with the Higgs field are of the order of the electroweak scale. While vacuum energy may be simply set to the value that would explain the late-time observations, this does not offer any insight into why it has this value.

In the above discussion it has been assumed that we can use the rules of flat spacetime QFT to calculate vacuum expectation values. However, the structure of QFT in flat spacetime is rooted in special properties of Minkowski space which curved spacetime does not share. The crux of the vacuum energy issue is the relation between the energy-momentum tensor and spacetime curvature, so we should consider how the situation changes when QFT is formulated consistently in curved spacetime.

In QFT based on axioms suited to curved spacetime, the vacuum energy of a massive free scalar field is arbitrary for non-zero mass, but necessarily vanishes for zero mass, in contrast to the flat spacetime theory, in which the vacuum energy is

always arbitrary [9, 10]. Vacuum expectation values are expected to depend non-analytically on the parameters of the theory, and it has been suggested that this could lead to vacuum energy much smaller than the natural scale of the theory, and that this might be relevant for present-day cosmic acceleration [10].

To elaborate further, we can propose the following scenario. Since the Standard Model Higgs field is massive, it might seem that the above result is not relevant. However, if there is no elementary Higgs field, the situation changes [4]. If it is also true for fermions and vector fields that vacuum energy vanishes for zero mass and there are no elementary scalars, so that all masses are dynamically generated, then this would explain why the vacuum energy is zero. (This is sometimes known as the "old" vacuum energy problem.) Two questions immediately arise. What happens when interactions are included, in particular those which dynamically generate masses and composite particles? And second, is it possible to understand the value of vacuum energy which would explain the observations? These issues turn out to be related.

It may be that as the symmetry which keeps particles at zero mass is dynamically broken, it can no longer enforce vanishing vacuum energy. Vacuum energy should thus vanish as coupling constants go to zero, the theory becomes free and symmetry is restored. Vacuum energy should also depend non-analytically on the couplings. For a coupling constant g, perhaps the simplest possibility is e^{-1/g^2}. In electroweak physics, the scale of particle masses is related to the Higgs vacuum expectation value $v \approx 246$ GeV. Taking $g^2 = \alpha \approx 1/137$, the vacuum energy density could then naively be expected to be

$$\rho_{\rm vac} \sim e^{-1/g^2} v^4 \approx (0.33 \text{ meV})^4 \ . \tag{1}$$

This is close to the value that would explain the observations, $\rho_{\rm vac} = 3H^2 M_{\rm Pl}^2 \Omega_\Lambda \approx (h/0.7)^2 (\Omega_\Lambda/0.7)\ (2.3 \text{ meV})^4$. The estimate is exponentially sensitive to inserting $e^2 = 4\pi\alpha$ instead of α and to the presence of other factors such as weak mixing angles, and the functional form is mere conjecture. Nevertheless, this does show that the scale which would explain late-time cosmological observations may emerge naturally from electroweak physics in curved spacetime.

We can outline steps to take to see whether the above conjecture holds any truth. First we should consider the vacuum energy of free fermions and vector bosons in curved spacetime QFT, then introduce a model of symmetry breaking and trace what happens to vacuum energy. Eventually, a full model of dynamical mass generation should be considered: realistic models tend to have a new scale in addition to the electroweak scale, and the dynamics can be rather involved [11].

It may not be necessary to deal with spacetime curvature. The key ingredient is the new set of axioms: this is not flat spacetime QFT, but the flat spacetime limit of curved spacetime QFT. The experimental success of QFT based on flat spacetime axioms shows that the curved spacetime theory must reduce extremely accurately to

the flat spacetime version as far as non-gravitational physics is concerned. However, when it comes to effects which are absent or ambiguous in the flat spacetime theory and to which particle physics experiments are not sensitive, the results can be very different.

Conclusions. If vacuum energy is the cause of the increased expansion rate at late times, its value is tiny compared to particle physics scales. A natural explanation would be that there is a symmetry which sets vacuum energy exactly to zero, and which is slightly broken. In the present scenario, the symmetry is related to particles having zero mass, and it is broken by the emergence of new effective degrees of freedom in dynamical breaking of electroweak symmetry. Vacuum energy is thus, like massive particles, an emergent phenomenon. Dynamical mass generation is not a small effect, but its impact on vacuum energy is diluted exponentially by the non-analytical dependence of vacuum expectation values on coupling constants.

The scenario requires that there are no massive elementary scalar fields, in particular that any Higgs particle is a composite, which is a qualitative prediction for collider physics. As vacuum energy is related to the spectrum of masses of particles at the electroweak scale, the vacuum energy problem is connected to the Higgs mass hierarchy problem and it may be possible to probe it at the LHC. It is also necessary that there are no particles with masses larger than the electroweak scale, or that the contribution of such particles to vacuum energy is correspondingly more suppressed. On the theoretical side, the dynamical generation of a mass scale and the choice of the axioms of QFT relates vacuum energy to the Millennium problem of defining Yang-Mills theory and understanding the mass gap [12].

Even if the idea turns out to be wrong, it shows that addressing the vacuum energy problem does not necessarily require delving into speculative physics distant from observations, and that there is little reason to abandon hope that a solution exists.

References

[1] P.J.E. Peebles and B. Ratra, *The Cosmological constant and dark energy*, *Rev. Mod. Phys.* **75** (2003) 559 [arXiv:astro-ph/0207347]

[2] H. Kragh, *Preludes to dark energy: Zero-point energy and vacuum speculations* [arXiv:1111.4623 [physics.hist-ph]]

[3] C.B. Collins and S.W. Hawking, *Why is the Universe isotropic?*, *Astrophys. J.* **180** (1973) 317

[4] S. Räsänen, *Backreaction: directions of progress* [arXiv:1102.0408 [astro-ph.CO]]

[5] T. Buchert and S. Räsänen, *Backreaction in late-time cosmology* [arXiv:1112.5335 [astro-ph.CO]]

[6] R.L. Jaffe, *The Casimir effect and the quantum vacuum*, *Phys. Rev.* **D72** (2005) 021301 [arXiv:hep-th/0503158]

[7] http://www.nobelprize.org/nobel_prizes/physics/laureates/2011/advanced.html

[8] S. Hollands and R.M. Wald, *Quantum field theory is not merely quantum mechanics applied to low energy effective degrees of freedom* *Gen. Rel. Grav.* **36** (2004) 2595 [arXiv:gr-qc/0405082]

[9] S. Hollands and R.M. Wald, *Axiomatic quantum field theory in curved spacetime*, *Commun. Math. Phys.* **293** (2010) 85 [arXiv:0803.2003 [gr-qc]]

[10] S. Hollands and R.M. Wald, *Quantum field theory in curved spacetime, the operator product expansion and dark energy*, *Gen. Rel. Grav.* **40** (2008) 2051, *Int. J. Mod. Phys.* **D17** (2009) 2607 [arXiv:0805.3419 [gr-qc]]

[11] K. Lane, *Two lectures on technicolor* [arXiv:hep-ph/0202255]
C.T. Hill and E.H. Simmons, *Strong dynamics and electroweak symmetry breaking*, *Phys. Rept.* **381** (2003) 235, Erratum: ibid. **390** (2004) 553 [arXiv:hep-ph/0203079].

[12] http://www.claymath.org/millennium/Yang-Mills_Theory/